Preference-based spectrum pricing in dynamic spectrum access networks

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Preference-Based Spectrum Pricing in Dynamic Spectrum Access Networks

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***Abstract*—With market-driven secondary spectrum trading, licensed users can receive benefits in terms of monetary rewards or various transmission services, thus setting a fair pricing structure by suitably defining spectrum quality characteristics and accurately addressing participant's requirement is a key issue. In this paper, we investigate the pricing-based spectrum access by casting the problem of spectrum pricing into a Hotelling game model according to spectrum quality diversity. Particularly, we first build a pricing system model where unused spectrum from primary systems with different qualities forms a spectrum pool and can be divided into a number of uniform channels. A secondary user purchases a channel for usage according to its selection preference which is closely related to the channel quality and spectrum evaluation. The secondary user not only needs to consider the channel's quality and price, but also the interference cost on primary system. Detailed analysis on the policy preference of both primary system and secondary buyer are provided. By forming a game problem of spectrum pricing between primary and secondary users, we apply the Hotelling game model to handle the interaction between the participants. Specifically, by fixing Nash equilibrium of the game, an iterative algorithm for spectrum pricing is proposed based on the distribution characteristics of secondary user's preference. Essential analysis for the existence and uniqueness of the Nash equilibrium along with algorithm's convergence conditions are provided. Numerical results are also supplemented to show the effectiveness of the proposed algorithm in ensuring spectrum owner's profit.**

***Index Terms*—Dynamic spectrum access, spectrum allocation, Nash equilibrium, iterative convergence**

I. Introduction

With the development of emerging new multimedia services, growing demands for spectrum resource cannot be fully satisfied by the latest multimedia transmission and broadband technique. The contradiction between congested available spectrum bands for new applications and underutilized allocated spectrum reveals the shortcomings of the current static spectrum allocation policy. Thus, dynamic spectrum access has been considered as a promising way to improve the utilization of scarce spectrum [1]-[5].

To make full use of the spectrum resources and realize dynamic spectrum sharing between primary and secondary users, many state-of-the-art methods have been studied and deployed in existing literatures [6]-[8]. One of the key challenges in dynamic spectrum access networks is to re-use spectrum holes such that primary networks are protected from interference while the quality of service (QoS) of the secondary users is guaranteed. The strategies of spectrum access and power allocation are widely explored to mitigate the interference on primary networks and smooth spectrum handover [9]-[13]. In underlay mode, the operation of secondary user is permitted as long as the interference caused by the secondary user does not affect the primary user's QoS, when they coexist in the networks. In overlay mode, secondary users opportunistically occupy the primary user's idle spectrum until they detect the primary activity again.

So far, market-based mechanisms have been investigated as a promising approach of dynamically assigning available spectrum to interested buyers. In particular, auction-based spectrum access has been considered as an efficient means of dynamically assigning available spectrum to potential secondary users [14]-[18]. Notably, the authors in [14] raise a general framework for truthful double spectrum auctions, which applies a novel winner determination and pricing mechanism to achieve truthfulness and other economic properties while significantly improving spectrum utilization. The authors in [15] design a secondary spectrum trading market when there are multiple sellers and buyers and propose a general framework for the trading market based on an auction mechanism. In [16], the authors analyze both sequential and concurrent auction mechanism for allocating the coordinated access band spectrum, which is divided into a fixed number of chunks. The allocation must satisfy a constraint that each bidder is allocated at most one chunk of the spectrum. Reference [17] considers a short-term secondary spectrum trading between one seller and multiple buyers in a hybrid spectrum market with both guaranteed contracts and spot transactions. In addition, [18] presents a secondary spectrum market where a primary license holder can sell access to its unused or under-used spectrum resources in the form of certain fine-grained spectrum-space-time unit. Specifically, the authors investigate auction mechanisms to allocate and price spectrum resources to maximize license holder's revenue.

Consider the spectrum dynamics and lack of centralized authority and smooth interaction, spectrum access strategy always needs to adapt to the node mobility, channel variations and dynamic wireless traffic based on locally observed information in a distributed manner. It is proved that, compared to the auction-based spectrum access, the pricing-based spectrum access incurs lower overhead and interaction [19]-[23]. In [19], the transmission rate of each secondary user is assumed to be a function of network congestion (like for TCP

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traffic) and the price per bandwidth unit. Primary operators sell spare bandwidth to secondary users, and set spectrum access prices to maximize their revenue. The authors in [20] propose a joint power/channel allocation scheme that improves the performance through a distributed pricing approach. In the scheme, a price-based iterative water-filling algorithm is designed, which enables secondary users to reach a good Nash equilibrium. Reference [21] presents a techno-economic analysis for regulated secondary access based on a base station-centric framework, where secondary users coexist with the subscribers, i.e., primary users, on a mutually exclusive basis. The framework is aimed at maximizing the localized spectrum utilization within the static spectrum licensed to the wireless service providers. Furthermre, reference [22] analyzes a price competition scenario by identifying a class of conflict graphs, in which the authors refer to as mean valid graphs, such that the conflict graphs of a large number of topologies that commonly arise in practice are mean valid. Besides, in reference [23], a hierarchical spectrum trading model is presented to analyze the interaction among WRAN service providers, TV broadcasters and WRAN users.

To improve the success of dynamic spectrum access, providing proper economic incentives to all parties involved is essential. In recent years, market-driven secondary spectrum trading has been widely investigated as a promising approach to address the incentive issue. However, there are still some challenges when spectrum trading is applied in practical complicated networks. Unlike traditional commodities, the availability of leasing spectrum is often not deterministic. Due to the uncertainty of primary user's activities, the spectrum owner usually cannot obtain the availability information in advance and may require to withdraw part of leasing spectrum stochastically. In this circumstance, how to establish a suitable and flexible contact to balance secondary user's interest and stabilize the market need to be further considered [24], [25]. Furthermore, most of the existing works are based on a basic assumption: The information interactions between the primary users and secondary users as well as the secondary users themselves are transparent and smooth, and the time, cost and transmit power involved in the course can be ignored [26], [27]. Thus, a specific and feasible interaction protocol used in the auction behavior should be investigated and formed. Besides, during dynamic spectrum allocation, the spectrum for sale will change with surrounding cell condition. It can be imaged that the channel suffered from lower interference levels owns higher marginal cost and is desired to reap more revenue for the primary system. Furthermore, the secondary user's individual demand and budget level are also different which leads to the diversity of spectrum selection preference. In this circumstance, the spectrum owner should design a detailed channel pricing mechanism to adapt to the development of spectrum quality and customer demand with the intention of profit maximum.

Market-based spectrum allocation has been constantly investigated in recent years, because of its incentive nature to best utilize the limited radio spectrum and match telecom operators' own interests. Generally, the study on the spectrum trading is unfolded in the following approaches: At first, in

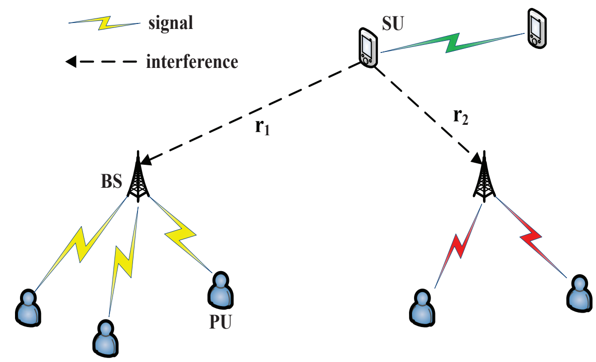

Fig. 1. System model for dynamic spectrum access networks. BS, PU and SU represent base station, primary user and secondary user, respectively.

order to achieve the demand balance between spectrum owners and secondary buyers, online auction with unknown or fixed spectrum supplies is proposed to enable reusability-driven spectrum allocation [28], [29]. Particularly, the truthful auction mechanism is the key issue during the course which attracts lots of attention. Secondly, joint channel and power allocation scheme is investigate to guarantee primary system's profit and suitably control the internet interference caused by the secondary customers [30], [31]. Third, pricing-based spectrum trading is designed to maximize the social welfare or spectrum utility efficiency where the total transmission capacity of the secondary or primary users attracts main concerns [32]. In this paper, we mainly investigate the impact of secondary user's spectrum preference on the channel pricing and system profit during the heterogenous spectrum trading. In this case, different user behavior characteristics in spectrum usage are discussed to improve the system's benefit. In our system model, we assume the unused spectrum of primary system constitutes a spectrum pool. Then, the idle spectrum with uniform bandwidth is divided into high-quality channel or low-quality channel for sale according to different interference levels suffered which are caused by adjacent cells or other secondary users. Thus, a specific pricing strategy appropriately revealing the diversity of spectrum quality and the supply-demand relationship of spectrum trading needs to be addressed naturally. On the other hand, a secondary user chooses one channel for usage in the light of its selection preference which is obviously affected by the emergence degree of spectrum demand and its available fund. Based on the distribution characteristics of secondary selection preference, we achieve a pricing solution for the primary system by using Hotelling model.

The following points highlight the main contributions of the paper:

- We classify the leased channels by spectrum quality and propose a system model to specifically describe the diversity of leased channels as well as secondary buyer's demands.
- A preference parameter is designed to formulate the satisfactory degree of the secondary user on the channels with unlike qualities. Detailed analysis on the effect of

trading preference for both primary system and secondary buyer is provided. To our knowledge, there are few research works investigating the impact of secondary user's preference on the spectrum pricing in dynamic access networks when primary system's spectrum is divided into various types.

- A suitable objective function decided by the secondary user's preference characteristic and desired marginal cost of the leased channel is proposed to build a Hotelling model. Besides, detailed undifferentiated preference pattern and spectrum exchange path with regard to supply-demand relationship and channel qualities are further provided.
- An iterative pricing algorithm is achieved by fixing the Nash equilibrium. The system model we proposed in this work approaches to the real commodity deal in daily life where the buyers always need to make a subtle balance between the product quality and price.
- Essential analysis and discussion on the existence of Nash equilibrium and the convergence conditions of the iterative algorithm are provided. Furthermore, corresponding analysis on the numerical results is provided to evaluate the interaction between the channel quality and spectrum pricing in dynamic access networks.

The rest of this paper is structured as follows: Section II describes the system model in dynamic spectrum access networks. The algorithm based on the Hotelling model for solving the problem of spectrum pricing is presented in Section III. Furthermore, the effects of selection preferences of primary system and secondary dealer on spectrum trading along with the proof for the Nash equilibrium and convergence characteristic of our proposal are also provided in Section III. Section IV presents the numerical results and analyses. Finally, Section V concludes this paper.

II. System Model

As the radio spectrum allocated to TV services remains largely unoccupied in many areas, the FCC initiates a change in the spectrum usage policy to allow unlicensed service to operate on the sub-900 MHz TV bands in which two kinds of systems including TV system and wireless microphone system are usually working [33]. In this paper, we thus consider a system model where two licensed user systems and numbers of unlicensed users coexist as shown in Fig. 1. Suppose the primary systems are working on a broad band and the unused spectrum can form a shared-use pool in which the overall available bands are divided into lots of uniform channels to facilitate subsequent sale. The channel qualities in the spectrum pool are not symmetrical due to different interference levels caused by random user locations. The channels in the spectrum pool are assumed to be orthogonal, and primary systems will select part of the unused channels to lease as shown in Fig. 2. We suppose every single secondary user can only purchase one channel for usage, and every buyer has its own selection preference. In this case, the spectrum suppliers are supposed to be fixed and the spectrum buyers cannot purchase spectrum resource in other ways. For primary

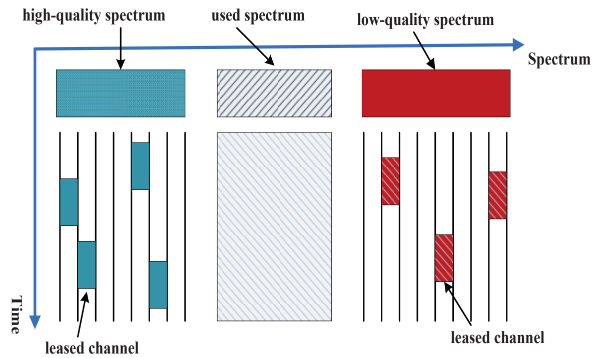

Fig. 2. A spectrum pool where two kinds of leased channels located

systems, it is essential to study a suitable pricing mechanism for maximizing own revenue according to the distribution characteristic of buyer's preference.

We assume the interference suffered by an immobile secondary terminal is not uniform when it uses a channel in the spectrum pool. In same bandwidth, a leased channel with high-quality spectrum can be expressed as C_h, and with low-quality spectrum as C_l, where $C_h > C_l > 0$. Parameter C_i denotes the channel capacity which can be given as

$$C_i = B\log_2(1+\frac{\rho_w}{I_{s_i}}), \tag{1}$$

where B is bandwidth, ρ_w is the power received by the secondary user and I_{s_i} is the interference suffered at the channel. In fixed bandwidth B and power ρ_w, different I_{s_i} mean diverse spectrum qualities. It can be imaged that the interference suffered by the secondary purchaser is mainly invoked by mobile terminals within the cell, adjacent cells even other secondary users unregistered.

In this case, we can cast the pricing problem into the model of duopoly competition where two suppliers, similar to high-quality spectrum and low-quality spectrum, competing in prices and products tend to attract more potential buyers which will finally lead to a dynamic balance. This idea became known among economists as the principle of minimum differentiation which first proposed by Hotelling [34]. In fact, a mature market is always controlled by only few huge corporations which likely comes into being a monopoly economy. Hotelling generalized Bertrand's model by considering different firms' locations in geographic space, later this model was more often interpreted as a model of product differentiation which is an important feature of actual business. In this paper, we also assume all the participants are rational with the intention of pursuing interest maximization. Furthermore, the primary systems expect to sell all of idle channels rather than the low-quality channels only, therefore an appropriate pricing strategy should be designed so as to attain a market equilibrium. Here, it should be mentioned that it is worthy of researching the heterogeneous channel pricing in this situation despite we only divide the idle spectrum into two types of channels. In common economic field, many enterprises cannot subdivide their product into many types in order to occupy the whole markets due to the limitation of enterprise scale and capital. In dynamic access networks, lots of primary users or small stations usually do not require dividing the spectrum very detailedly with plenty of computational cost, especially when the interference condition within the cell is not very complicated.

III. Channel pricing

A. User Preference

In this work, we consider the spectrum price includes two parts, one is the basic value representing variant spectrum qualities, and the other is the interference pricing. The interference within the primary systems will increase with the upgrade of system load. Furthermore, we consider the secondary utility function can contain three parts, including transmission rates, channel price and increased interference level. For the secondary consumer, the only intention of its transaction behavior is to obtain right amount of spectrum usage with minimum budget. The reduction part of the secondary user's utility is mainly caused by its monetary cost. Besides, we consider the interference increase invoked by the secondary customer on the primary seller in details, since it will lead to the increase of the spectrum pricing eventually. Thus, secondary user's utility function can be given as

$$U_s = \kappa \times \theta \times C_i - p_i - \varepsilon I_i \tag{2}$$

where C_i denotes the spectrum quality, p_i is channel pricing, and I_i is the interference level caused by the secondary purchaser on primary system. κ and ε are monetary coefficients, and θ is user preference parameter. Here, we introduce parameter θ to describe user's demand preference. Define θ locates at the region $[\overline{\theta}, \underline{\theta}]$ with probability distribution function (PDF) $g(\theta)$. It can be envisioned that the user with more budget fund will prefer to select a high-quality channel. In general, secondary users should make a rational choice between channel quality and cost. In this case, the preference parameter θ plays a key role to influence its final decision. Here, we define a non-preference coefficient θ_b to describe the balancing state for the secondary buyer. When $\theta = \theta_b$, it means the secondary user has no preference for any kind of channels since equal cost function can be attained at this moment. Thus, there is the following equation

$$U_S = {U_S}' \Rightarrow \kappa\times\theta_b\times C_l - p_l - \varepsilon I_l = \kappa\times\theta_b\times C_h - p_h - \varepsilon I_h \tag{3}$$

Then, we have

$$\theta_b = \frac{p_h - p_l + \varepsilon(I_h - I_l)}{\kappa(C_h - C_l)} \tag{4}$$

where I_i denotes the interference caused by the spectrum buyer on the primary receiver which can be expressed as $I_i = g_{ij}\rho_{w_j}$. Parameter g_{ij} which is positive represents the path gain (not including fading) from the jth transmitter to the ith receiver. It can be expressed as $g_{ij} = h_{ij}c_{ij}$, where $h_{ij} = A/r_{ij}^{\alpha}$ and c_{ij} is the correlation coefficient. r_{ij} denotes the distance between terminal i and terminal j. A is a constant gain, and suppose $\alpha = 2$. In the analysis below, we assume g_{ij} is constant, and does not change much with time. Rewriting (4), we can obtain

$$\theta_b = \frac{p_h - p_l + \varepsilon A\rho_{w_j}(c_{hj}/r_{hj}^2 - c_{lj}/r_{lj}^2)}{\kappa(C_h - C_l)} \tag{5}$$

ρ_{w_j} is the transmit power of the secondary user. When secondary user's preference parameter satisfies $\theta > \theta_b$, it prefers to choose a high-quality channel. On the other hand, when $\theta < \theta_b$, it is likely to purchase a low-quality channel for usage.

Secondary buyer's preference diversity on different kinds of channels will affect its own selection as well as the channel's price. Besides, for a primary system, its channel pricing can be decided directly by the scarcity degree of current idle channels and potential market evaluation on its channel quality. We thus define parameter θ_1 to denote the primary system's preference on the leased channels which implies an anticipated price. Similarly, for the secondary users, they need to make a judgement whether or not to accept the channel price according to their capital budget and spectrum demand. We use parameter θ_2 to express the secondary preference of owing capital so as to make full use of the limited money to exchange the access authority. Then, we analyze the impact of the secondary preference on spectrum trading for both primary systems and secondary users.

Prior to the channel trading, we suppose the initial number of primary idle channels to be i_0 and the secondary user's budget as r_0 as shown in Fig. 3. We further consider the idle channel number and acquired revenue of the primary system change to be i and r after the deal. Therefore, the channel and capital quantity occupied by the secondary user are $i_0 - i$ and $r - r_0$ after the spectrum trading, respectively. Thus, in a rectangular coordinate system as shown in Fig. 3, any coordinate (r, i) can represent a pattern of spectrum trading in the rectangle $0 \le r \le r_0, 0 \le i \le i_0$. In Fig. 3, the undifferentiated preference curves of the primary system are presented. The curves show a downward trend with the progress of the spectrum trading, since the primary system needs to deliver part of idle channels if it wishes to receive profit. Meanwhile, the secondary user obtains the spectrum authority by paying the fee within its budget.

We utilize undifferentiated curves to depict the preference degree of the primary system on unused idle channels and reaping profit. If the primary system can receive similar satisfaction with either occupying i_2 idle channels and r_2 revenue at v_2 in Fig. 3, or preserving i_1 idle channels and r_1 revenue at v_1, we consider there is no difference for the primary system at point v_1 and point v_2. Another word, compared with v_1, it is also acceptable for the primary system to decrease channel number in content of $(i_1 - i_2)$ in exchange for receiving system profit in $(r_2 - r_1)$. Therefore, as shown in Fig. 3, all the points with same satisfaction degree in v_1 and v_2 can form an undifferentiated curve MN, and the points with higher satisfaction degree for the primary system exist in another curve M_1N_1. It can be envisioned many undifferentiated curves for the primary system can coexist in the figure, and we express the curve family as

$$f(r, i) = \chi_1 \tag{6}$$

where χ_1 is the satisfaction degree. When χ_1 increases, the curves move upper-right. Furthermore, as idle channel's quality upgrades, the curves will decrease slowly than before as shown in Fig. 4 since the primary system wishes to achieve more profit when better channels have been provided for sale. Otherwise, the curves will get down fast in condition of worse channel quality.

Proposition 1. The primary system's undifferentiated curves are monotone decreasing, concave and disjoint.

Proof: Firstly, we analyze the monotone decreasing characteristic of the undifferentiated curves. In this coordinate system shown in Fig. 3, abscissa denotes the revenue received and ordinate denotes the idle channel number. For the primary system, the channel resource is its bargaining counter in this deal. If the primary systme wants to attain the goal of revenue increase, it needs to decrease the number of idle channels as an essential cost with the progress of the trading. Hence, the curves should be monotone decreasing.

Then, we analyze the convexity and concavity of the undifferentiated curves. For a common trader, the quantity of goods has apparent affection on the pricing. Large numbers of commodities likely lead to a lower price. On the contrary, price rising may be triggered by lack of commodity. Due to the scarcity of future usable spectrum and blooming band demand of various wireless application, the channel pricing tends to upgrade with the decrease of usable channel number. In general, the bargainer participating in the spectrum trading would rather pay more $\triangle y$ in exchange for comparatively less $\triangle x$ in condition that it occupies few x which is shown as the red triangle in Fig. 5. Similar conclusion is that the primary system tends to raise the price when it had leased most of idle channels and reaped sufficient profit shown as the blue triangle in Fig. 5. An essential condition for the deal realization is the mutual satisfactory for both participants, and it is understandable that the primary system adjusts pricing strategy according to its available commodity quantity.

On the other hand, it can be envisioned that primary system's undifferentiated curves could be affected apparently by the spectrum supply-demand relationship during the trading course. Primary system may choose a steady pricing strategy to stabilize market expectation and promote product sales. In this case, primary system's preference curve should be a straight line as shown in Fig. 6. In addition, when the supply-demand relationship of the spectrum market changes and the supply exceeds demand, which means there are large numbers of leasing channels and less secondary demands, the primary system's preference curve is more likely to become upper-concave. In current cognitive radio networks, the lack of spectrum resource and rapid increase of secondary users will lead to the preference performances shown in Fig. 5 where spectrum price should unavoidablly increases when leased channel becomes less.

Last, we prove no joint point can exist in the undifferentiated curves by proof of contradiction. If there is a joint point m in the coordinate system which means two undifferentiated curves pass through the point, then we have distinct satisfaction degrees at point m for the primary system. The conclusion is not suitable for one primary system since same utility cost

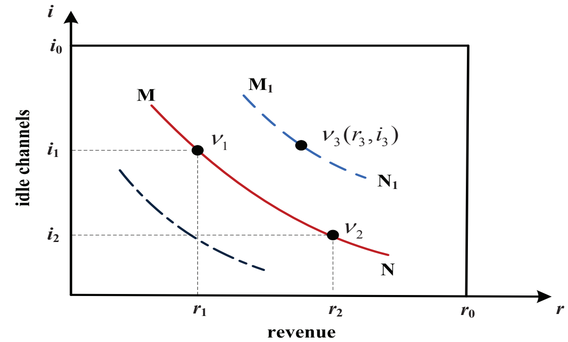

Fig. 3. Undifferentiated preference curves of primary system. i denotes idle channel number and r denotes the revenue received.

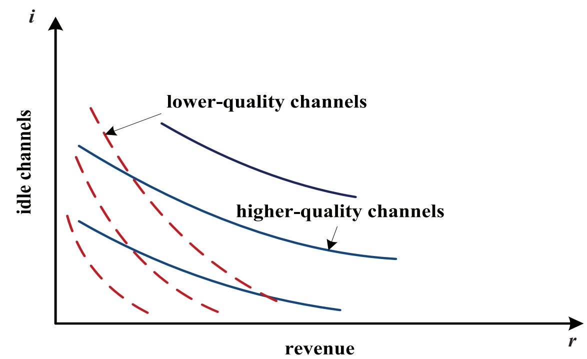

Fig. 4. Undifferentiated curves of primary system with different channel qualities.

can be only reached at one point in this coordinate system. Therefore, the undifferentiated preference curves for a same system is disjoint. ■

Similarly, the secondary users have another undifferentiated curve family in the spectrum trading which can be expressed as following

$$\varphi(r, i) = \chi_2 \tag{7}$$

No matter what the equations of f, φ are, every participant owns preference curves in the light of its demand on the spectrum. Furthermore, in order to obtain satisfactory exchange pattern for both parts, we mix their undifferentiated curves together as shown in Fig. 7, where the coordinate system and curves marked in red line denote the performance of the secondary customers and the black line represents the primary system. In the figure, the primary system's undifferentiated curves $f(r, i) = \chi_1$ and the secondary user's curves family $\varphi(r, i) = \chi_2$ can be depicted with different coordinates origins. We connect the cross points of two curve families and mark them by dotted line AB as shown in Fig. 7.

Proposition 2. The trading points satisfying both participants should locate at the curve AB, which can be called exchange path.

Proof: If the spectrum trading happens at another point v' outside the exchange path AB as shown in Fig. 7, we can suppose the interaction point is v generated by the cross

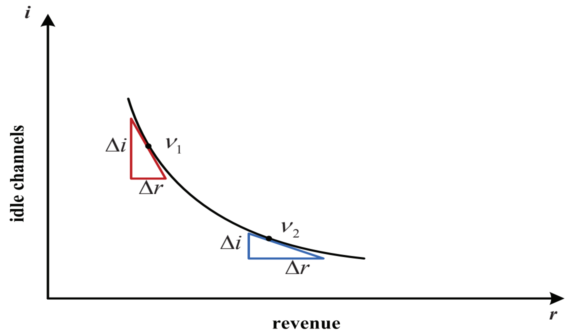

Fig. 5. Concave curves.

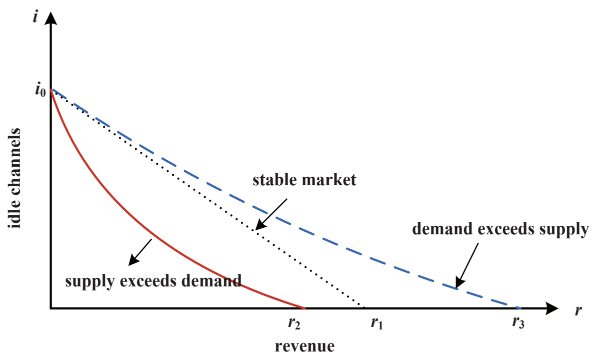

Fig. 6. Undifferentiated curves of primary system with different pricing strategies.

of the undifferentiated curve through v' and AB. Then, the primary system obtains same satisfaction degree in v and v' while the satisfaction degree of secondary user on v is higher than that of v'. The satisfaction degrees for both parts are obviously different, and the secondary user would rather stay at point v than v' in order to save the money. Consequently, the participant involved in the trading cannot reach a balancing satisfaction degree at the point v' to finish the deal.

B. Pricing Game

Having analyzed the relationship between user's preference and channel price, then we discuss the pricing game for both the high-quality channel and low-quality channel of the primary system in this subsection by using Hotelling game model. For the seller, the quantity and price of the commodity are the most important issues which are both affected by market demand. In this case, the demand functions for high-quality and low-quality channels can be given as

$$\begin{aligned} D_h(p_h,p_l) &= N\int_{\theta_b}^{\overline{\theta}} g(\theta)d\theta = N[1-G(\theta_b)] \\ &= N\{1-G[\frac{p_h-p_l+\varepsilon(I_h-I_l)}{\kappa(C_h-C_l)}]\} \end{aligned} \tag{8}$$

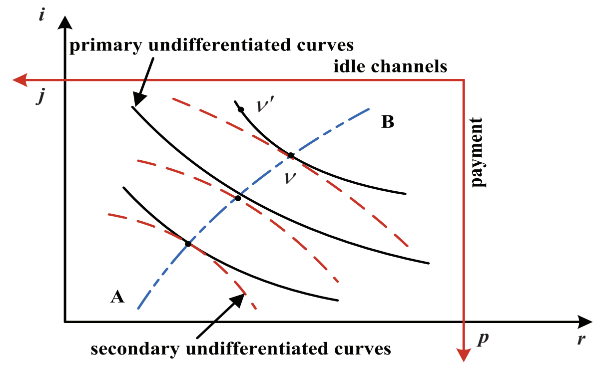

Fig. 7. Exchange path.

$$\begin{aligned} D_l(p_h,p_l) &= N\int_{\underline{\theta}}^{\theta_b} g(\theta)d\theta = NG(\theta_b) \\ &= NG[\frac{p_h-p_l+\varepsilon(I_h-I_l)}{\kappa(C_h-C_l)}] \end{aligned} \tag{9}$$

where N is the number of secondary users, and $g(\theta)$ is the probability distribution function. It is clear that $0 < D_h < N$, $0 < D_l < N$. Suppose the cost functions of the licensed systems are $S_h = s_h D_h(p_h,p_l)$ and $S_l = s_l D_l(p_h,p_l)$, whereas the cost of high-quality channel is large than the cost of low-quality channel denoted as $0 < s_l < s_h$. According to (8) and (9), we can obtain the pricing functions as

$$p_h = f_h(q_h,q_l) = C_l F(1-\frac{q_h}{N}-\frac{q_l}{N}) + (C_h-C_l)F(1-\frac{q_h}{N}) \tag{10}$$

$$p_l = f_l(q_h,q_l) = C_l F(1-\frac{q_h}{N}-\frac{q_l}{N}) \tag{11}$$

where $F(\bullet)$ is the inverse function of $G(\theta)$, and $q_{h,l}$ are the corresponding channel number. Due to $F' = \frac{1}{G'(\theta)} > 0$, we have $F(1-\frac{q_h}{N}-\frac{q_l}{N}) < F(1-\frac{q_h}{N})$. Furthermore, since $0 < F(1-\frac{q_h}{N}-\frac{q_l}{N}) < 1$, there are $0 < p_h < C_h$, $0 < p_l < C_l$.

The product quantity competition of the licensed system can be formulated by Cournot bargaining model [35], [36], and the profit functions of the high-quality channel and low-quality channel in this case can be expressed as

$$\pi_H^C(q_h,q_l) = q_h[C_l F(1-\frac{q_h}{N}-\frac{q_l}{N}) + (C_h-C_l)F(1-\frac{q_h}{N}) - C_h] \tag{12}$$

$$\pi_L^C(q_h,q_l) = q_l[C_l F(1-\frac{q_h}{N}-\frac{q_l}{N}) - C_l] \tag{13}$$

In this paper, we mainly want to investigate the spectrum allocation strategy in perspective of pricing bargaining instead of quantity competition. For primary systems, they can lease part of temporarily unused spectrum for reaping monetary incomes or preserve the channels for future potential applications. Here, we assume the marginal cost for every single channel to be $M_i = \mu C_i$, where μ is a cost coefficient. Due to sacrificing the reuse right of the leased spectrum for future appplications, primary systems need to take the cost into account. Thus, the profit functions for high-quality and low-quality channels can be given as

$$\pi_h(p_h,p_l) = (p_h - M_h)D_h = N(p_h-\mu C_h)\{1-G[\frac{p_h-p_l+\varepsilon(I_h-I_l)}{\kappa(C_h-C_l)}]\} \tag{14}$$

$$\pi_l(p_h,p_l) = (p_l - M_l)D_l = N(p_l-\mu C_l)G[\frac{p_h-p_l+\varepsilon(I_h-I_l)}{\kappa(C_h-C_l)}] \tag{15}$$

For given preference region $[\underline{\theta},\overline{\theta}]$, different PDFs of the preference parameter will have an apparent impact on the final pricing scheme of the primary systems. When the secondary users have abundant budget to purchase the channels or put more emphases on the spectrum quality than spectrum price, it can be predicted that they will prefer to select a high-quality channel with a relatively expensive cost. In this case, we assume the PDF of parameter θ complies with the linear distribution in region $[\underline{\theta},\overline{\theta}]$, therefore the utility functions for the leased channels can be expressed as

$$\begin{aligned}\pi_h(p_h,p_l) &= N(p_h-\mu C_h)\int_{\theta_b}^{\overline{\theta}} g(\theta)d\theta \\ &= N(p_h-\mu C_h)\int_{\theta_b}^{\overline{\theta}} \tau\theta d\theta \\ &= \frac{1}{2}N\tau(p_h-\mu C_h)(\overline{\theta}^2-\theta_b^2) \\ &= \frac{1}{2}N\tau(p_h-\mu C_h)\times \\ \{\overline{\theta}^2 - &\frac{(p_h-p_l)^2+\varepsilon(I_h-I_l)[\varepsilon(I_h-I_l)+2(p_h-p_l)]}{\kappa^2(C_h-C_l)^2}\}\end{aligned} \tag{16}$$

$$\begin{aligned}\pi_l(p_h,p_l) &= N(p_l-\mu C_l)\int_{\underline{\theta}}^{\theta_b} g(\theta)d\theta \\ &= N(p_l-\mu C_l)\int_{\underline{\theta}}^{\theta_b} \tau\theta d\theta \\ &= \frac{1}{2}N\tau(p_l-\mu C_l)(\theta_b^2-\underline{\theta}^2) \\ &= \frac{1}{2}N\tau(p_l-\mu C_l)\times \\ \{&\frac{(p_h-p_l)^2+\varepsilon(I_h-I_l)[\varepsilon(I_h-I_l)+2(p_h-p_l)]}{\kappa^2(C_h-C_l)^2} - \underline{\theta}^2\}\end{aligned} \tag{17}$$

In addition, when preference parameter θ is an uniform distributed variable located in region $[\overline{\theta},\underline{\theta}]$, the profit functions can be obtained as

$$\begin{aligned}\pi_h(p_h,p_l) &= N(p_h-M_h)\int_{\theta_b}^{\overline{\theta}} g(\theta)d\theta \\ &= \frac{N(p_h-\mu C_h)}{\overline{\theta}-\underline{\theta}}[\overline{\theta}-\frac{p_h-p_l+\varepsilon(I_h-I_l)}{\kappa(C_h-C_l)}]\end{aligned} \tag{18}$$

$$\begin{aligned}\pi_l(p_h,p_l) &= N(p_l-M_l)\int_{\underline{\theta}}^{\theta_b} g(\theta)d\theta \\ &= \frac{N(p_l-\mu C_l)}{\overline{\theta}-\underline{\theta}}[\frac{p_h-p_l+\varepsilon(I_h-I_l)}{\kappa(C_h-C_l)}-\underline{\theta}]\end{aligned} \tag{19}$$

In this paper, we mainly focus on the situation where the secondary selection preference complies with the linear distribution in $[\overline{\theta},\underline{\theta}]$, and similar conclusions can also be achieved in case of uniformed distribution of θ. When secondary users prefer high-quality channel even though they have to afford expensive cost, primary system needs to design a suitable price mechanism attracting the consumers to obtain higher profits. We thus consider the primary systems are rational whose main objective is to pursue profit maximization. Based on the necessary conditions for Nash equilibrium, the optimal channel pricing at step $k+1$ can be derived from (16) and (17) as following

$$\begin{aligned}p_h^{k+1} &= p_h^k + \frac{\partial\pi_h(p_h,p_l)}{\partial p_h} \\ &= p_h^k + \frac{N\tau}{2}\{\overline{\theta}^2 - \frac{(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon(I_h-I_l))}{\kappa^2(C_h-C_l)^2} + \\ &\frac{\varepsilon^2(I_h-I_l)^2+2(p_h^k-\mu C_h)(p_h^k-p_l^k+\varepsilon(I_h-I_l))}{\kappa^2(C_h-C_l)^2}\}\end{aligned} \tag{20}$$

$$\begin{aligned}p_l^{k+1} &= p_l^k + \frac{\partial\pi_l(p_h,p_l)}{\partial p_l} \\ &= p_l^k + \frac{N\tau}{2}\{\frac{(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon(I_h-I_l))}{\kappa^2(C_h-C_l)^2} + \\ &\frac{\varepsilon^2(I_h-I_l)^2+2(p_l^k-\mu C_l)(p_l^k-p_h^k-\varepsilon(I_h-I_l))}{\kappa^2(C_h-C_l)^2} - \underline{\theta}^2\}\end{aligned} \tag{21}$$

Hence, after setting initial pricing p_h^0, p_l^0, we can obtain the optimal pricing strategy by numbers of iterative operations. We will give essential proofs for the existence of the Nash equilibrium and the iterative convergence characteristic later. Then, by ascertaining the equilibrium point of the spectrum pricing, we can obtain the optimal system profits as following

$$\begin{aligned}\frac{\partial\pi_h(p_h,p_l)}{\partial p_h} &= 0 = \overline{\theta}^2 - \\ &\frac{(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon')+\varepsilon'^2+2(p_h^k-2\mu C_h)(p_h^k-p_l^k+\varepsilon')}{\kappa'}\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{\partial\pi_l(p_h,p_l)}{\partial p_l} &= 0 = \\ &\frac{(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon')+\varepsilon'^2+2(p_l^k-2\mu C_l)(p_l^k-p_h^k-\varepsilon')}{\kappa'} - \underline{\theta}^2\end{aligned} \tag{23}$$

where $\varepsilon' = \varepsilon(I_h-I_l)$ and $\kappa' = \kappa^2(C_h-C_l)^2$. Rewriting (22) and (23), we have

$$(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon')+\varepsilon'^2+2(p_h^k-2\mu C_h)(p_h^k-p_l^k+\varepsilon') = \kappa'\overline{\theta}^2 \tag{24}$$

$$(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon')+\varepsilon'^2+2(p_l^k-2\mu C_l)(p_l^k-p_h^k-\varepsilon') = \kappa'\underline{\theta}^2 \tag{25}$$

Combining (24) and (25), we can obtain

$$\begin{aligned}&(p_h^k-p_l^k)(p_h^k-p_l^k+2\varepsilon')+\varepsilon'^2+(p_h^k-p_l^k+\varepsilon')\times \\ &(p_h^k-p_l^k-2\mu C_h+2\mu C_l) = \frac{\kappa'(\overline{\theta}^2+\underline{\theta}^2)}{2}\end{aligned} \tag{26}$$

$$(p_h^k - p_l^k + \varepsilon')(p_h^k + p_l^k - 2\mu C_h - 2\mu C_l) = \frac{\kappa'(\overline{\theta}^2 - \underline{\theta}^2)}{2} \quad (27)$$

Substituting $p_h^k - p_l^k$ and $p_h^k + p_l^k$ by X and Y, we can get $p_h^k = (X+Y)/2$ and $p_l^k = (X-Y)/2$. Rewriting (26) and (27), we have

$$\begin{cases} (X+\varepsilon')(Y - 2\mu C_h - 2\mu C_l) = \frac{\kappa'(\overline{\theta}^2 + \underline{\theta}^2)}{2} \\ X(X+2\varepsilon') + \varepsilon'^2 + (X+\varepsilon')(X - 2\mu C_h + 2\mu C_l) = \frac{\kappa'(\overline{\theta}^2 - \underline{\theta}^2)}{2} \end{cases} \quad (28)$$

Solving (28) and discarding imaginary solutions, we can get

$$\begin{cases} X = \frac{[(3\varepsilon'+\mu') - 8\varepsilon'\mu' + 8\theta']^{1/2} - 3\varepsilon' - \mu'}{4} \\ Y = \frac{\theta''}{X+\varepsilon'} + 2\mu C_h + 2\mu C_l \end{cases} \quad (29)$$

where $\mu' = 2\mu C_l - 2\mu C_h$, $\theta' = \kappa'(\overline{\theta}^2 - \underline{\theta}^2)/2$ and $\theta'' = \kappa'(\overline{\theta}^2 + \underline{\theta}^2)/2$. Then, we can ascertain p_h^k, p_l^k by $p_h^k = (X+Y)/2$ and $p_l^k = (X-Y)/2$. Substituting p_h, p_l in (16) and (17), we can achieve the optimal system profits. Supplemental comments for the pricing mechanism are given as **Remarks** below.

Remark 1. For a primary system, the definition of the marginal cost for every single channel which is denoted as $M_i = \mu C_i$ contains an assumption $\mu C_h > \mu C_l \Rightarrow M_h > M_l$. Due to $C_h > C_l$, the assumption implies the high-quality channels consume more potential costs for the primary system. In another word, it is considered that the primary system would rather choose the idle channels with high quality for selling to reap high revenues than reserving these ideal channels for future applications. Accordingly, the primary system is willing to afford the potential marginal cost in order to increase current capital incomes.

From another perspective, in the condition that the marginal cost difference of high-quality and low-quality channels can be ignored, we assume the marginal cost denoted by $M_h = M_l = \mu(C_h + C_l)/2$. Then, when the preference parameter is a linear distributed variable in region $[\underline{\theta}, \overline{\theta}]$, the system profit functions can be expressed as

$$\begin{aligned} \pi_h(p_h, p_l) &= N[p_h - \frac{\mu(C_h + C_l)}{2}] \int_{\theta_b}^{\overline{\theta}} g(\theta) d\theta \\ &= N[p_h - \frac{\mu(C_h + C_l)}{2}] \int_{\theta_b}^{\overline{\theta}} \tau\theta d\theta \\ &= \frac{1}{2} N\tau [p_h - \frac{\mu(C_h + C_l)}{2}] \times \\ &\{\overline{\theta}^2 - \frac{(p_h - p_l)^2 + \varepsilon(I_h - I_l)[\varepsilon(I_h - I_l) + 2(p_h - p_l)]}{\kappa^2 (C_h - C_l)^2}\} \end{aligned} \quad (30)$$

$$\begin{aligned} \pi_l(p_h, p_l) &= N[p_l - \frac{\mu(C_h + C_l)}{2}] \int_{\underline{\theta}}^{\theta_b} g(\theta) d\theta \\ &= N[p_l - \frac{\mu(C_h + C_l)}{2}] \int_{\underline{\theta}}^{\theta_b} \tau\theta d\theta \\ &= \frac{1}{2} N\tau [p_l - \frac{\mu(C_h + C_l)}{2}] \times \\ &\{\frac{(p_h - p_l)^2 + \varepsilon(I_h - I_l)[\varepsilon(I_h - I_l) + 2(p_h - p_l)]}{\kappa^2 (C_h - C_l)^2} - \underline{\theta}^2\} \end{aligned} \quad (31)$$

Taking the derivation, the optimal channel pricing at step $k+1$ can also be obtained.

Remark 2. For a primary system, like an usual producer, it is essential to guarantee the cost difference between two kinds of channels can be covered by the price difference which can be expressed as

$$M_h - M_l = \mu(C_h - C_l) \leq P_h^* - P_l^* \quad (32)$$

If the customers cannot afford the price difference, an alternative choice for the primary system is to keep the high-quality channels idle for future possible applications.

Remark 3. When secondary user's preference complies to normal distribution, we cannot obtain an closed-form equation due to lacking complete integrability. When the preference parameter's distribution function is complicated, we should judge whether or not the function is integrable.

Theorem 1. For given function ξ_i, if there exists $\alpha \geq 0$ making $\sup_{i \in N} E|\xi_i|^\alpha < \infty$, then $\xi_i, i \in N$ is consistent integrable.

Proof: If we have $\sup_{i \in N} E|\xi_i|^\alpha < \infty$, let $\lambda_\varepsilon = \sup_{i \in N} E|\xi_i|^\alpha / \delta_\varepsilon < \infty$. Then, in condition of $\lambda > \lambda_\varepsilon$, there is

$$P(|\xi_i| \geq \lambda) \leq E|\xi_i|/\lambda \leq \sup_{i \in N} E|\xi_i|^\alpha / \lambda < \delta_\varepsilon \quad (33)$$

where ξ_i is the selection preference of secondary user i. As a result, $E|\xi_i| I_{\{|\xi_i| \geq \lambda\}} < \varepsilon$ works for any $t \in N$, which means

$$\sup_{i \in T} E|\xi_i|^\alpha I_{\{|\xi_i| \geq \lambda\}} \leq \varepsilon, \quad \alpha \geq 0 \quad (34)$$

where for every $\omega \in \Omega$

$$I_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \in A^C := \Omega \setminus A \end{cases} \quad (35)$$

Furthermore, for any $A \in \Re$ and $\lambda > 0$, we have

$$\begin{aligned} &\sup_{i \in N} E|\xi_i|^\alpha I_A \\ &= \sup_{i \in N} (E|\xi_i|^\alpha I_{A \bigcap \{|\xi_i| < \lambda\}} + E|\xi_i|^\alpha I_{A \bigcap \{|\xi_i| \geq \lambda\}}) \\ &\leq \lambda P(A) + \sup_{i \in N} E|\xi_i|^\alpha I_{\{|\xi_i| \geq \lambda\}} \end{aligned} \quad (36)$$

If $\{\xi_i, i \in N\}$ is consistent integrable, we can fix $A = \Omega$ and make λ_0 large enough so that $\sup_{i \in N} E|\xi_i|^\alpha I_{\{|\xi_i| \geq \lambda_0\}} < 1$, then there is $\sup_{i \in N} E|\xi_i|^\alpha \leq \lambda_0 + 1 < \infty$. In addition, for any $\varepsilon > 0$, allow λ_ε large enough to make $\sup_{i \in N} E|\xi_i|^\alpha I_{\{|\xi_i| \geq \lambda_\varepsilon\}} < \varepsilon/2$. Let $\delta_\varepsilon = \varepsilon/(2\lambda_\varepsilon)$, for any $A \in \Re$, only if $P(A) < \delta_\varepsilon$, we can obtain

$$\sup_{i \in N} E|\xi_i|^\alpha I_A \leq \lambda_\varepsilon P(A) + \varepsilon/2 < \varepsilon \quad (37)$$

Then, the sufficient conditions for consistent integrability can be met.

C. Nash Equilibrium and Iterative Convergence

In this section, essential proofs for the existence of the Nash equilibrium and the convergence of the proposed iterative algorithm are supplied.

Theorem 2. Having obtained utility function $u_i = \pi_i$ and strategy function $s_i = p_i$ as given above, there is a pure Nash equilibrium for the spectrum pricing game.

Proof: According to Debreu's equilibrium existence theorem [37], for the strategy function s_i and utility function u_i, there is a pure strategy Nash equilibrium if the following sufficient conditions can be met: (1) s_i is a nonempty and compact subset in limited Euclidean space. (2) For strategy combination S, u_i is continuous and concave.

First, for strategy combination S, define $S : \Sigma \to \Sigma$ is the Cartesian direct product of s_i and Σ_i is a simplex with dimension $|s_i|$. s_i is a compact subset in limited Euclidean space if and only if s_i are uniformly bounded and equicontinuous functions. If we want to prove s_i are uniformly bounded, then $\forall \varepsilon_1$, it needs to certify that $\exists \delta = \delta(\varepsilon_1)$ makes $\forall \varphi \in F$, thus

$$|\varphi(x_1) - \varphi(x_2)| < \varepsilon_1 \quad (when\ \rho(x_1, x_2) < \delta) \tag{38}$$

where $\rho(x, y) \triangleq \max\limits_{a \le t \le b} |x(t) - y(t)|$ and $\varphi(x_i) \Rightarrow s_i$. Since $\varepsilon_1/3$ subset of F is a finite set $N(\varepsilon_1/3) = \{\varphi_1, \varphi_2, \cdots, \varphi_n\}$, according to continuity, $\exists \delta = \delta(\varepsilon/3)$, then in condition of $\rho(x_1, x_2)$, there is

$$|\varphi_i(x_1) - \varphi_i(x_2)| < \varepsilon_1/3 \quad (i = 1, 2, \cdots, n) \tag{39}$$

Due to $\forall \varphi \in F$, $\varphi_i \in N(\varepsilon/3)$ making $d(\varphi, \varphi_i) < \varepsilon/3$, there is

$$\begin{aligned} &|\varphi(x) - \varphi(x')| \\ &\le |\varphi(x) - \varphi_i(x)| + |\varphi_i(x) - \varphi_i(x')| + |\varphi_i(x) - \varphi(x')| \\ &\le 2d(\varphi, \varphi_i) + |\varphi_i(x) - \varphi_i(x')| < \varepsilon_1 \quad (when\ \rho(x, x') < \delta) \end{aligned} \tag{40}$$

where $d(u, v) = \max |u(x) - v(x)|$. Furthermore, it is obvious that s_i is continuous and differentiable, therefore we can conclude that s_i is a nonempty and compact subset in limited Euclidean space.

Second, according to the definition of concave function $f(tx + (1+t)y) \ge tf(x) + (1-t)f(y)$, u_i is concave if $\pi_i^{''} \le 0$. For the high-quality channels, we needs

$$\pi_h^{''} = \frac{N\tau(2p_h - \mu C_h)}{2\kappa^2(C_h - C_l)^2} < 0 \tag{41}$$

Then, we can have

$$p_h < \mu C_h/2 \tag{42}$$

Appropriate parameter setting will meet this condition. Thus, we prove a pure Nash equilibrium can exist in this case which is also unique [38].

Theorem 3. The iterative pricing algorithm we proposed denoted as (20) and (21) can be convergent in precondition of appropriate parameter settings.

Proof: Rewriting the iterative pricing algorithm denoted as (20) and (21) in matrix form, we have

$$P^{k+1} = U^{-1}WP^k + U^{-1}b, \quad k = 0, 1, 2, \cdots \tag{43}$$

where

$$U^{-1}W = \begin{pmatrix} 1+\frac{N\tau(2p_h-\mu C_h)}{\kappa'} & \frac{N\tau}{\kappa'}(2\mu-2p_l+2\varepsilon') \\ \frac{N\tau}{\kappa'}(2p_h-4p_l+2\varepsilon'-2\mu C_l) & 1+\frac{N\tau}{\kappa'}(6p_l-4p_h-2\mu C_l) \end{pmatrix} \tag{44}$$

and

$$U^{-1}b = \begin{pmatrix} \frac{N\tau}{2}(\overline{\theta}^2 - \frac{\varepsilon'^2-2\mu\varepsilon' C_h}{\kappa'}) \\ \frac{N\tau}{2}(\frac{\varepsilon'^2+2\mu\varepsilon' C_l}{\kappa'} - \underline{\theta}^2) \end{pmatrix} \tag{45}$$

where $\kappa' = \kappa^2(C_h - C_l)^2, \varepsilon' = \varepsilon(I_h - I_l)$.

According to Reference [39], if there is $\rho(U^{-1}W) < 1$, then the iterative algorithm will be convergent. Based on the Jacobi iteration denoted in (43), the Jacobi matrix can be expressed as (46). Then, the maximal eigenvalue of the iterative matrix can be obtained as (47). Hence, the pricing iterative algorithm for the primary systems finally obtained can be convergent in condition of appropriate parameter settings by making $\rho(U^{-1}W) < 1$.

IV. Numerical Results

In this section, we provide numerical results to show the effectiveness of the proposed pricing algorithm. Moreover, the iterative convergence and Nash equilibrium of the solution are also evaluated. In this case, we consider the spectrum trading is performed in cognitive radio networks in which the licensed users dominate the spectrum and the interference from adjacent cells can be ignored. Secondary users without participating the spectrum trading are not allowed to access the licensed spectrum. For the iterative pricing algorithm, the initial spectrum pricing for both low-quality and high-quality channels is $p_{h,l}^{(0)} = 0.01$. We set monetary coefficients $\kappa = 10, \varepsilon = 0.5$ and cost coefficient $\mu = 0.2$, $0.1 \le C_l \le C_h \le 3$(bps), $1 \le \underline{\theta} < \overline{\theta} \le 10$, $\triangle I = 0.001$mW, $\tau = 0.2$. The initial channel prices can be set randomly, and we select a small initial value in this case. Since the proposed algorithm is convergent in proper parameter settings, the pricing curve will converge to same value regardless of the initial point. We further set common initial monetary coefficients along with cost coefficients, and the impacts of different κ and μ on the channel pricing will provided later in this section. The main intention of the secondary users participating in the spectrum trading is to achieve channel authority, thus we make $\kappa > \varepsilon$ to put the emphasis of secondary utility on transmission capacity. C_h and C_l is set in unit bandwidth, and there is $C_h > C_l$. Besides, as the interference power is usually much lower than transmit power of secondary terminal, we fix a small $\triangle I$. Different effects of parameter τ on channel pricing are also provided in subsequent tests.

In Fig. 8, the changes of high-quality channel' pricing with different channel quality parameters are given. We can obtain from Fig. 8 that the price of high-quality channel increases obviously with growing high-quality channel coefficient C_h in which another significant quality coefficient C_l is fixed to be 0.3. Except for C_h, the other parameters are constant during the course which have been given above. As shown in Fig. 8, the pricing algorithms converge in a relative fast speed in which the proposed algorithm can achieve a steady result after

$$
\begin{aligned}
U^{-1}W &= \begin{pmatrix} \frac{\partial p_h^{k+1}}{\partial p_h} & \frac{\partial p_h^{k+1}}{\partial p_l} \\ \frac{\partial p_l^{k+1}}{\partial p_h} & \frac{\partial p_l^{k+1}}{\partial p_l} \end{pmatrix} \\
&= \begin{pmatrix} 1+\frac{N\tau(2p_h-\mu C_h)}{\kappa'} & \frac{N\tau}{\kappa'}(2\mu-2p_l+2\varepsilon') \\ \frac{N\tau}{\kappa'}(2p_h-4p_l+2\varepsilon'-2\mu C_l) & 1+\frac{N\tau}{\kappa'}(6p_l-4p_h-2\mu C_l) \end{pmatrix}
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
&\rho(U^{-1}W) = \lambda^* = 1 - \frac{N\tau\psi}{2\kappa'} + \frac{1}{2}\sqrt{(2+\frac{N\tau\psi}{\kappa'})^2 - 4[(1+\frac{N\tau a'}{\kappa'})(1+\frac{N\tau d'}{\kappa'}) - \frac{N^2\tau^2 b'c'}{\kappa'}]} \\
&\quad where \ \ \psi = p_h + 4\varepsilon' - p_l - \mu(C_h - C_l), \ \kappa' = \kappa^2(C_h - C_l)^2, \\
&\quad a' = \mu C_h - 3p_h - 2p_l - 2\varepsilon', \ b' = \mu C_h - 2p_h - p_l + \varepsilon', \\
&\quad c' = p_h + 2p_l - \mu C_l + \varepsilon', \ d' = 2p_h + 3p_l - \mu C_l - 2\varepsilon', \\
&\quad \varepsilon' = \varepsilon(I_h - I_l).
\end{aligned}
\tag{47}
$$

about ten iterations. Furthermore, the high-quality channel's variation trend tells that it is predictable for the primary system that the improvement of channel quality will increase the channel marginal cost and attract more potential consumers. In this case, a condition we assumed is the preference demand of the secondary users complies with the linear distribution which means the spectrum buyers prefer to select high-quality channels than low-quality channels. Therefore, the primary system can perform the spectrum trading in the assumption that the secondary users would rather pay more money for higher-quality channels. Then, any increase of channel's quality is expected to stimulate the consumption. However, the primary system also needs to afford more marginal cost when the spectrum quality is improved, thus it is reasonable for the primary system to increase the price so as to compensate higher potential cost.

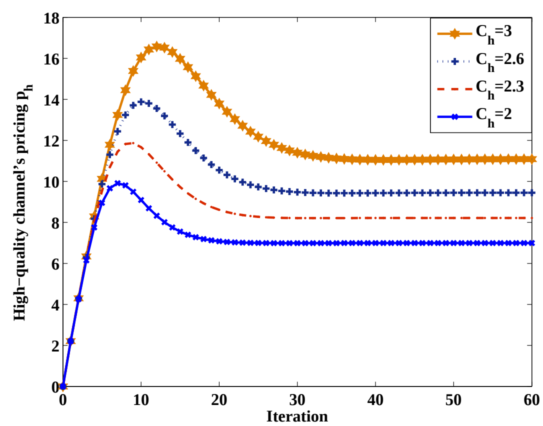

Fig. 8. High-quality channel's pricing with different C_h

In Fig. 9, the pricing performance of high-quality channel is further shown with growing C_l where the other parameters keep unchanged as Fig. 8. Descending C_l means a continuous quality decline for low-quality channel. In this case, there are more reasons for secondary users with linear preference parameters to pursue high-quality channels. Thus, more demands for the high-quality channels will lead to price increase. However, we can also obtain from Fig. 9 that the pricing increase of high-quality channel becomes insignificant even though parameter C_l continues decreasing to a lower level, which means the acceptable price of the high-quality channel for the secondary buyer is not unlimited when only the parameter C_l decreases. We also add an iterative pricing curve with $C_h = 2, C_l = 0.3$ and another iterative curve with $C_h = 2.3, C_l = 0.3$ to Fig. 9. We can conclude from Fig. 8 and Fig. 9 that the increase of quality gap between high-quality channel and low-quality channel will bring up the price of high-quality channels, when the selection preference of secondary users is linear distributed.

We also show the low-quality channel pricing with different channel coefficients C_h and C_l in Fig. 10 and Fig. 11, respectively. In Fig. 10, we set a fixed low-quality channel coefficient $C_l = 0.8$ with variable C_h to testify the impacts on the pricing of low-quality channels. When the quality of high-quality channel increases which means a relative quality descending for the low-quality channel, rational secondary users will tend to make a choice with higher income. Hence, it can explain the variation trend of low-quality channel's pricing with variable C_h. Besides, in Fig. 11, we set the high-quality channel coefficient as $C_h = 2.5$. As shown in Fig. 11, the pricing of low-quality channel will have a normal increase with improved channel quality. We also observe that the low-quality channel's pricing cannot decrease indefinitely even the coefficient C_l becomes very low, since a bottom price for the low-quality channel should be able to cover the marginal cost of the primary system.

Other parameters also have obvious impacts on the channel pricing. Fig. 12 shows the results in conditions of variant monetary coefficients κ where solid-line curves represent the pricing of high-quality channel and dotted line curves represent the pricing of low-quality channel. We set the algorithm's coefficients as $\kappa = 10, \varepsilon = 0.5$ and cost coefficient $\mu = 0.2$, $C_l = 0.3$, $C_h = 2$, $\underline{\theta} = 1$, $\overline{\theta} = 10$, $\triangle I = 0.001$. When monetary coefficient κ increases from 3 to 16, both of the high-quality and low-quality channels' pricing increases. A rise of the monetary coefficient κ means the increase of the

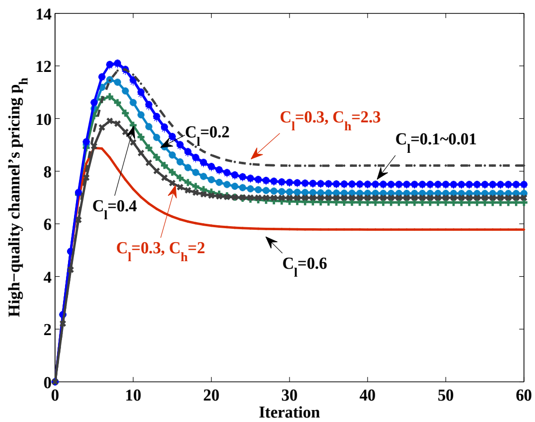

Fig. 9. High-quality channel's pricing with different C_l

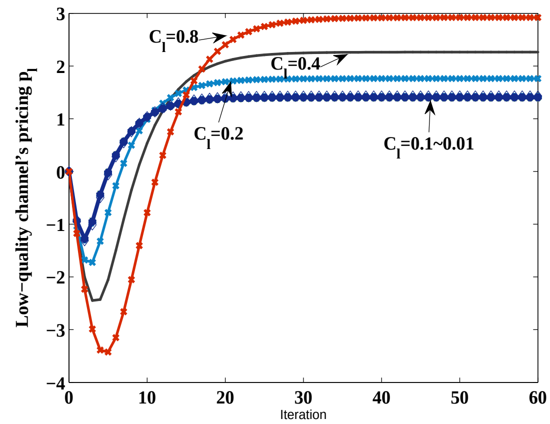

Fig. 11. Low-quality channel's pricing with different C_l

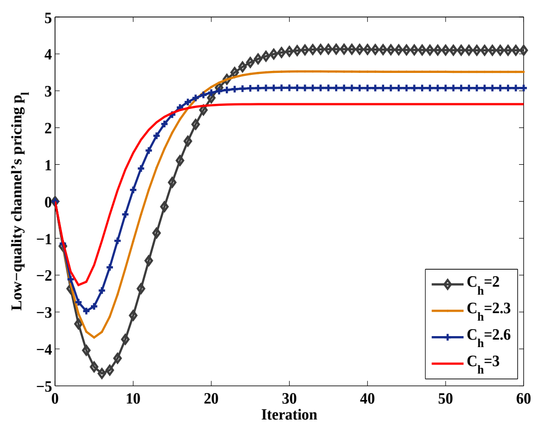

Fig. 10. Low-quality channel's pricing with different C_h

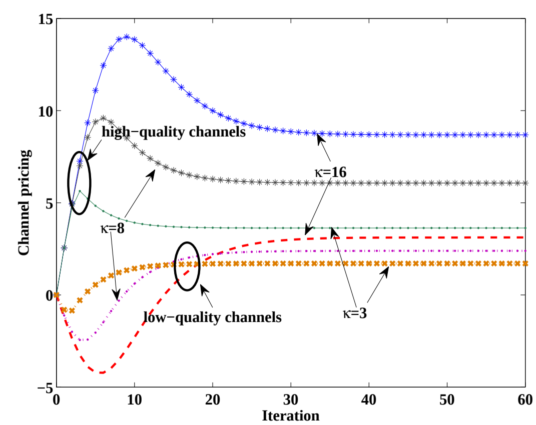

Fig. 12. Channel pricing with variant monetary coefficients κ

idle spectrum's evaluation value. For the secondary users, a larger monetary coefficient can increase its utility function as shown in (2), thus the secondary users can afford higher price with better profit expectation.

In Fig. 13, the effects of the high-quality channel's pricing with different convergent speeds are presented. Here, we set the algorithm parameters as $\kappa = 10, \varepsilon = 0.5$ and cost coefficient $\mu = 0.2$, $C_l = 0.3$, $C_h = 2$, $\underline{\theta} = 1$, $\overline{\theta} = 10$, $\triangle I = 0.001$. And the coefficient τ is changing from 0.1 to 0.9. The convergence speeds of our proposed iterative algorithm can be adjusted by tuning τ without changing other parameters. τ can be regarded as the regulatory factor for convergent speed. By adjusting τ , we can accelerate the algorithm convergence without changing the pricing value. Also, a divergence state of the iterative algorithm is given in Fig. 14. We just raise τ to 1.5, and the other parameters remain unchanged. The iterative pricing algorithm becomes divergent when τ is too high. As we discussed in Section III, unsuitable parameter settings cannot guarantee the algorithm's convergence. A careful selection on the pricing parameters is needed to obtain a stable and ideal outcome.

Furthermore, Fig. 15 and Fig. 16 show the changes of the system profits with different spectrum pricing. Other parameters are settled as $\kappa = 10, \varepsilon = 0.5$, $C_l = 0.15$, $C_h = 3.5$, $\underline{\theta} = 1$, $\overline{\theta} = 10$, $N = 5$ and $\triangle I = 0.001$. We adjust μ from 0.5 to 1.5. As shown in Fig. 15, the curves which represent the profits of high-quality channels behave as convex function with maximum values. The peaks of the curves mean the maximum system profits. We can obtain from the optimal value when the primary system's spectrum pricing reaches the Nash equilibrium point, which is similar to real market that over price will discourage customer's demand and lead to the decrease of system's income in the end. Additionally, Fig. 16 shows that the system profit of low-quality channels is less than that of high-quality channels in same channel quantity. Since the secondary user's spectrum preference is linear distributed, the system profits become concentrated in the high-quality channels. We can also obtain from Fig. 15 and Fig. 16 that the channel pricing becomes higher with increasing cost coefficients μ, since high cost coefficient means the primary systems have to afford more marginal cost. Thus, in this condition, the primary systems need a high channel price to guarantee their profits.

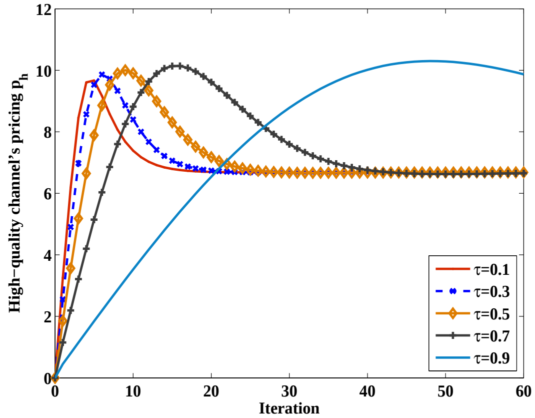

Fig. 13. Different convergence speeds

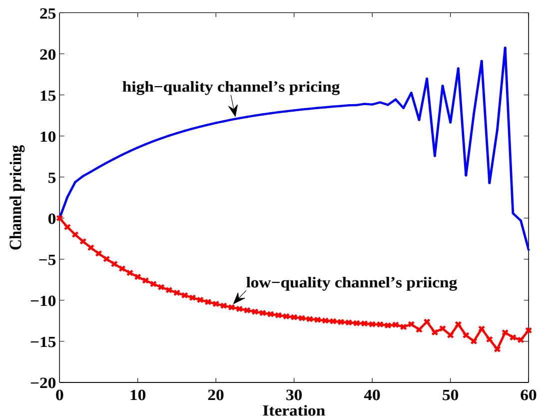

Fig. 14. Divergence of the pricing algorithm

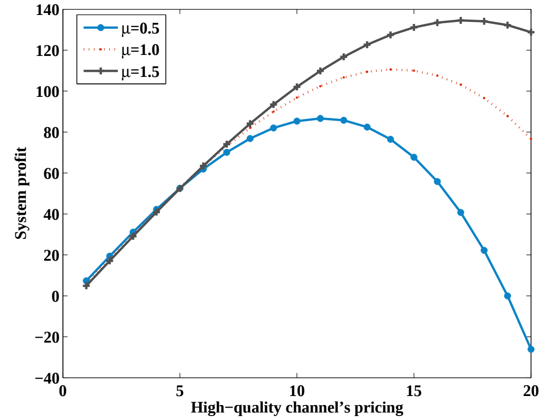

Fig. 15. High-quality pricing with different cost coefficients μ

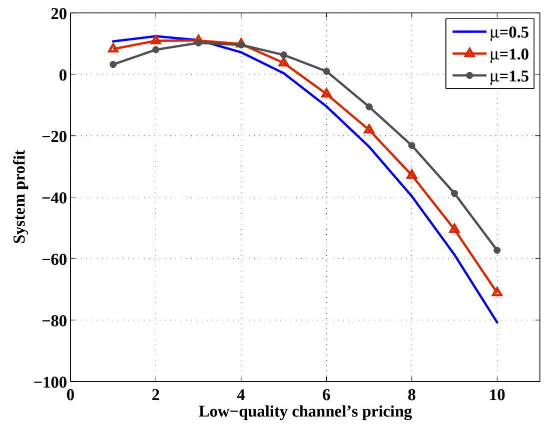

Fig. 16. Low-quality pricing with different cost coefficients μ

V. Conclusion

In this paper, a pricing-based spectrum access mechanism has been designed to solve the problem of spectrum allocation in dynamic access networks where lots of unused spectrums with different qualities constitute a spectrum pool and need to be suitably classified and priced. In order to accurately describe the spectrum quality diversity, we have introduced the concepts of interference difference caused by adjacent cells or other unlicensed secondary users and a preference parameter to represent secondary user's selection intention. Secondary user chooses an idle channel for usage according to its selection preference. We have further gave the analysis and discussion on the effects of the selection preference of both primary system and secondary buyer. A Hotelling game model which is fit for describing the market with product diversity is adopted to pursue the pricing algorithm. The system model we proposed in this work approaches to the real commodity deal in daily life where the customers always need to make a rational balance between product quality and price. Essential analysis and proof on the existence of Nash equilibrium and the convergence characteristics of the iterative algorithm are also provided. In future research work, the primary spectrum pricing in other conditions where the selection preference of the secondary user complies to more complicated distribution needs to be deeply investigated. Besides, we will consider to further distinguish primary spectrum by more kinds of qualities and study the corresponding trading preference of the secondary users along with the appropriate spectrum pricing strategy.

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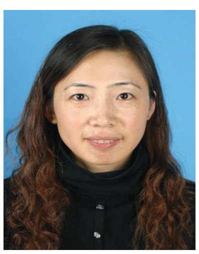

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